Индивидуальные домашние задания

***ИДЗ-1* Вычисление частных производных**

1 Найти область определения функций:

1.1 $z = 3xy/(2x-5y)$.
1.2 $z = \arcsin(x-y)$.
1.3 $z = \sqrt{y^2 - x^2}$.
1.4 $z = \ln(4 - x^2 - y^2)$.
1.5 $z = 2/(6 - x^2 - y^2)$.
1.6 $z = \sqrt{x^2 + y^2 - 5}$.
1.7 $z = \arccos(x+y)$
1.8 $z = 3x + y/(2 - x + y)$.
1.9 $z = \sqrt{9 - x^2 - y^2}$.
1.10 $z = \ln(x^2 + y^2 - 3)$.
1.11 $z = \sqrt{2x^2 - y^2}$.
1.12 $z = 4xy/(x - 3y + 1)$.
1.13 $z = \sqrt{xy}(x^2 + y^2)$.
1.14 $z = \arcsin(x/y)$.
1.15 $z = \ln(y^2 - x^2)$.
1.16 $z = x^3y/(3 + x - y)$.
1.17 $z = \arccos(x + 2y)$.
1.18 $z = \arcsin(2x - y)$.
1.19 $z = \ln(9 - x^2 - y^2)$.
1.20 $z = \sqrt{3 - x^2 - y^2}$.
1.21 $z = 1/\sqrt{x^2 + y^2 - 5}$.
1.22 $z = \dfrac{4x+y}{2x-5y}$.
1.23 $z = \dfrac{\sqrt{3x-2y}}{x^2+y^2+4}$.
1.24 $z = 5/(4 - x^2 - y^2)$.
1.25 $z = \ln(2x - y)$.
1.26 $z = 7x^3y/(x - 4y)$.
1.27 $z = \sqrt{1 - x - y}$.
1.28 $z = e^{\sqrt{x^2+y^2-1}}$.
1.29 $z = 1/(x^2 + y^2 - 6)$.
1.30 $z = 4xy/(x^2 - y^2)$.

2 Найти частные производные и частные дифференциалы функций:

2.1 $z = \ln(y^2 - e^{-x})$.
2.2 $z = \ln(\sqrt{xy} - 1)$.
2.3 $z = \arcsin\sqrt{xy}$.
2.4 $z = \arcsin(2x^3y)$.

2.5 $z = \mathrm{arctg}(x^2 + y^2)$.
2.6 $z = \mathrm{arctg}(x^2/y^3)$.
2.7 $z = \cos(x^3 - 2xy)$.
2.8 $z = \cos(x - \sqrt{xy^3})$.
2.9 $z = \sin\sqrt{y/x^3}$.
2.10 $z = \sin\dfrac{x+y}{x-y}$.
2.11 $z = \mathrm{tg}(x^3 + y^2)$.
2.12 $z = \mathrm{tg}\dfrac{2x - y^2}{x}$.
2.13 $z = \mathrm{ctg}\sqrt{xy^3}$.
2.14 $z = \mathrm{ctg}\sqrt{\dfrac{x}{x-y}}$.
2.15 $z = e^{-x^2+y^2}$.
2.16 $z = e^{-\sqrt{x^2+y^2}}$.
2.17 $z = \ln(3x^2 - y^4)$.
2.18 $z = \ln(3x^2 - y^2)$.
2.19 $z = \arccos(y/x)$.
2.20 $z = \arccos(x - y^2)$.
2.21 $z = \mathrm{arcctg}(xy^2)$.
2.22 $z = \mathrm{arcctg}\dfrac{x^3}{y}$.
2.23 $z = \cos\sqrt{x^2 + y^2}$.
2.24 $z = \cos\dfrac{x-y}{x^2+y^2}$.
2.25 $z = \sin\sqrt{x - y^3}$.
2.26 $z = \sin\sqrt{\dfrac{y}{x+y}}$.
2.27 $z = \mathrm{tg}(x^3y^4)$.
2.28 $z = e^{-(x^3+y^3)}$.
2.29 $z = \mathrm{ctg}(3x - 2y)$.
2.30 $z = e^{2x^2 - y^5}$.

3 Вычислить с точностью до двух знаков после запятой значения частных производных функции $f(x,y,z)$ в точке $M_0(x_0, y_0, z_0)$:

3.1 $f(x,y,z) = z/\sqrt{x^2+y^2}$, $M_0(0,-1,1)$.

3.2 $f(x,y,z) = \ln(x + \dfrac{y}{2z})$, $M_0(1,2,1)$.

3.3 $f(x,y,z) = (\sin x)^{yz}$, $M_0(\dfrac{\pi}{6},1,2)$.

3.4 $f(x,y,z)=\ln(x^3+2y^3-z^3)$, $M_0(2,1,0)$.

3.5 $f(x,y,z)=\dfrac{x}{\sqrt{y^2+z^2}}$, $M_0(1,0,1)$.

3.6 $f(x,y,z)=\ln\cos(x^2y^2+z)$, $M_0(0,0,\frac{\pi}{4})$.

3.7 $f(x,y,z)=27\sqrt[3]{x+y^2+z^3}$, $M_0(3,4,2)$.

3.8 $f(x,y,z)=\operatorname{arctg}(xy^2+z)$, $M_0(2,1,0)$.

3.9 $f(x,y,z)=\arcsin(x^2/y-z)$, $M_0(2,5,0)$.

3.10 $f(x,y,z)=\sqrt{z}\sin(y/x)$, $M_0(2,0,4)$.

3.11 $f(x,y,z)=y/\sqrt{x^2+z^2}$, $M_0(-1,1,0)$.

3.12 $f(x,y,z)=\operatorname{arctg}(xz/y^2)$, $M_0(2,1,1)$.

3.13 $f(x,y,z)=\ln\sin(x-2y+z/4)$, $M_0(1,1/2,\pi)$.

3.14 $f(x,y,z)=\dfrac{y}{x}+\dfrac{z}{y}-\dfrac{x}{z}$, $M_0(1,1,2)$.

3.15 $f(x,y,z)=\dfrac{1}{\sqrt{x^2+y^2-z^2}}$, $M_0(1,2,2)$.

3.16 $f(x,y,z)=\ln(x+y^2)-\sqrt{x^2z^2}$, $M_0(5,2,3)$.

3.17 $f(x,y,z)=\sqrt{z}x^y$. $M_0(1,2,4)$.

3.18 $f(x,y,z)=\dfrac{-z}{\sqrt{x^2+y^2}}$, $M_0(\sqrt{2},\sqrt{2},\sqrt{2})$.

3.19 $f(x,y,z)=\ln(x^3+\sqrt[3]{y}-z)$, $M_0(2,1,8)$.

3.20 $f(x,y,z)=z/(x^4+y^2)$, $M_0(2,3,25)$.

3.21 $f(x,y,z)=8\sqrt[5]{x^3+y^2+z}$, $M_0(3,2,1)$.

3.22 $f(x,y,z)=\ln(\sqrt[5]{x}+\sqrt[4]{y}-z)$, $M_0(1,1,1)$.

3.23 $f(x,y,z)=-2x/\sqrt{y^2+z^2}$, $M_0(3,0,1)$.

3.24 $f(x,y,z)=ze^{-(x^2+y^2)/2}$, $M_0(0,0,1)$.

3.25 $f(x,y,z)=\dfrac{\sin(x-y)}{z}$, $M_0(\frac{\pi}{2},\frac{\pi}{3},\sqrt{3})$.

3.26 $f(x,y,z)=\sqrt{z}\ln(\sqrt{x}+\sqrt{y})$, $M_0(4,1,4)$.

3.27 $f(x,y,z)=\dfrac{xz}{x-y}$, $M_0(3,1,1)$.

3.28 $f(x,y,z)=\sqrt{x^2+y^2-2xy\cos z}$, $M_0(3,4,\frac{\pi}{2})$.

3.29 $f(x,y,z)=ze^{-xy}$, $M_0(0,1,1)$.

3.30 $f(x,y,z)=\arcsin(x\sqrt{y})-yz^2$, $M_0(0,4,1)$.

4 Найти полный дифференциал первого порядка функций:

4.1 $z=2x^3y-4xy^3$.

4.2 $z=xy^4-3x^2y+1$.

4.3 $z=x^2y\sin x-3y$.

4.4 $z=\ln(x+xy-y^2)$.

4.5 $z=\operatorname{arctg}x+\sqrt{y}$.

4.6 $z=2x^2y^2+x^3-y^3$.

4.7 $z=\arcsin(xy)-3xy^2$.

4.8 $z=\sqrt{3x^2-2y^2+5}$.

4.9 $z=5xy^4+2x^2y^7$.

4.10 $z=\arcsin\dfrac{x+y}{x}$.

4.11 $z=\cos(x^2-y^2)+x^3$.

4.12 $z=\operatorname{arcctg}(x-y)$.

4.13 $z=\ln(3x^2-2y^2)$.

4.14 $z=\sqrt{3x^2-y^2+x}$.

4.15 $z=5xy^2-3x^3y^4$.

4.16 $z=y^2-3xy-x^4$.

4.17 $z=\arcsin(x+y)$.

4.18 $z=\arccos(x+y)$.

4.19 $z=\operatorname{arctg}(2x-y)$.

4.20 $z=\ln(y^2-x^2+3)$.

4.21 $z=7x^3y-\sqrt{xy}$.

4.22 $z=2-x^3-y^3+5x$.

4.23 $z=\sqrt{x^2+y^2-2xy}$.

4.24 $z=7x-x^3y^2+y^4$.

4.25 $z=e^{x+y-4}$.

4.26 $z=e^{y-x}$.

4.27 $z=\cos(3x+y)-x^2$.

4.28 $z=\operatorname{arctg}(2x-y)$.

4.29 $z=\operatorname{tg}(x+y)/(x-y)$.

4.30 $z=\operatorname{ctg}(y/x)$

5 Вычислить с точностью до двух знаков после запятой значение производной сложной функции $u = u(x, y)$, $x = x(t)$, $y = y(t)$, в точке $t = t_0$:

5.1 $u = e^{x-2y}$, $x = \sin t$, $y = t^3$, $t_0 = 0$.

5.2 $u = \ln(e^x + e^{-y}$, $x = t^2$, $y = t^3$, $t_0 = -1$.

5.3 $u = y^x$, $x = \ln(t-1)$, $y = e^{t/2}$, $t_0 = 2$.

5.4 $u = e^{y-2x+2}$, $x = \sin t$, $y = \cos t$, $t_0 = \pi/2$.

5.5 $u = x^2 e^y$, $x = \cos t$, $y = \sin t$, $t_0 = \pi$.

5.6 $u = \ln(e^x + e^y)$, $x = t^2$, $y = t^3$, $t_0 = 1$.

5.7 $u = x^y$, $x = e^t$, $y = \ln t$, $t_0 = 1$.

5.8 $u = e^{y-2x}$, $x = \sin t$, $y = t^3$, $t_0 = 0$.

5.9 $u = x^2 e^{-y}$, $x = \sin t$, $y = \sin^2 t$, $t_0 = \pi/2$.

5.10 $u = \ln(e^{-x} + e^y)$, $x = t^2$, $y = t^3$, $t_0 = -1$.

5.11 $u = e^{y-2x-1}$, $x = \cos t$, $y = \sin t$, $t_0 = \pi/2$.

5.12 $u = \arcsin(x/y)$, $x = \sin t$, $y = \cos t$, $t_0 = \pi$.

5.13 $u = \arccos(2x/y)$, $x = \sin t$, $y = \cos t$, $t_0 = \pi$.

5.14 $u = x^2/(y+1)$, $x = 1-2t$, $y = \mathrm{arctg} t$, $t_0 = 0$.

5.15 $u = x/y$, $x = e^t$, $y = 2 - e^{2t}$, $t_0 = 0$.

5.16 $u = \ln(e^{-x} + e^{-2y})$, $x = t^2$, $y = \frac{1}{3}t^3$, $t_0 = 1$.

5.17 $u = \sqrt{x + y^2 + 3}$, $x = \ln t$, $y = t^2$, $t_0 = 1$.

5.18 $u = \arcsin(x^2/y)$, $x = \sin t$, $y = \cos t$, $t_0 = \pi$.

5.19 $u = y^2/x$, $x = 1-2t$, $y = 1 + \mathrm{arctg} t$, $t_0 = 0$.

5.20 $u = \frac{y}{x} - \frac{x}{y}$, $x = \sin t$, $y = \cos t$, $t_0 = \frac{\pi}{4}$.

5.21 $u = \sqrt{x^2 + y + 3}$, $x = \ln t$, $y = t^2$, $t_0 = 1$.

5.22 $u = \arcsin\frac{x}{2y}$, $x = \sin t$, $y = \cos t$, $t_0 = \pi$.

5.23 $u = \frac{x}{y} - \frac{y}{x}$, $x = \sin 2t$, $y = tg^2 t$, $t_0 = \frac{\pi}{4}$.

5.24 $u = \sqrt{x + y + 3}$, $x = \ln t$, $y = t^2$, $t_0 = 1$.

5.25 $u = y/x$, $x = e^t$, $y = 1 - e^{2t}$, $t_0 = 0$.

5.26 $u = \arcsin(2x/y)$, $x = \sin t$, $y = \cos t$, $t_0 = \pi$.

5.27 $u = \ln(e^{2x} + e^y)$, $x = t^2$, $y = t^4$, $t_0 = 1$.

5.28 $u = \mathrm{arctg}(x + y)$, $x = t^2 + 2$, $y = 4 - t^2$, $t_0 = 1$.

5.29 $u = \sqrt{x^2 + y^2 + 3}$, $x = \ln t$, $y = t^3$, $t_0 = 1$.

5.30 $u = \mathrm{arctg}(xy)$, $x = t + 3$, $y = e^t$, $t_0 = 0$.

6 Вычислить с точностью до двух знаков после запятой значения частных производных функции $z(x, y)$, заданной неявно, в точке $M_0(x_0, y_0, z_0)$:

6.1 $x^3 + y^3 + z^3 - 3xyz = 4$, $M_0(2,1,1)$.

6.2 $x^2 + y^2 + z^2 - xy = 2$, $M_0(-1,0,1)$.

6.3 $3x - 2y + z = xz + 5$, $M_0(2,1,-1)$.

6.4 $e^z + x + 2y + z = 4$, $M_0(1,1,0)$.

6.5 $x^2 + y^2 + z^2 - z - 4 = 0$, $M_0(1,1,-1)$.

6.6 $z^3 + 3xyz + 3y = 7$, $M_0(1,1,1)$.

6.7 $\cos^2 x + \cos^2 y + \cos^2 z = \frac{3}{2}$, $M_0(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4})$.

6.8 $e^{z-1} = \cos x \cos y + 1$, $M_0(0, \frac{\pi}{2}, 1)$.

6.9 $x^2 + y^2 + z^2 - 6x = 0,\ M_0(1,2,1).$

6.10 $xy = z^2 - 1,\ M_0(0,1,-1).$

6.11 $x^2 - 2y^2 + 3z^2 - yz + y = 2,\ M_0(1,1,1).$

6.12 $x^2 + y^2 + z^2 + 2xz = 5,\ M_0(0,2,1).$

6.13 $x\cos y + y\cos z + z\cos x = \pi/2,\ M_0(0,\pi/2,\pi).$

6.14 $3x^2y^2 + 2xyz^2 - 2x^3z + 4y^3z = 4,\ M_0(2,1,2).$

6.15 $x^2 - 2y^2 + z^2 - 4x + 2z + 2 = 0,\ M_0(1,1,1).$

6.16 $x + y + z + 2 = xyz,\ M_0(2,-1,-1).$

6.17 $x^2 + y^2 + z^2 - 2xz = 2,\ M_0(0,1,-1).$

6.18 $e^z - xyz - x + 1 = 0,\ M_0(2,1,0).$

6.19 $x^3 + 2y^3 + z^3 - 3xyz - 2y - 15 = 0,\ M_0(1,-1,2).$

6.20 $x^2 - 2xy - 3y^2 + 6x - 2y + z^2 - 8z + 20 = 0,\ M_0(0,-2,2).$

6.21 $x^2 + y^2 + z^2 = y - z + 3,\ M_0(1,2,0).$

6.22 $x^2 + y^2 + z^2 + 2xy - yz - 4x - 3y - z = 0,\ M_0(1,-1,1).$

6.23 $x^2 - y^2 - z^2 + 6z + 2x - 4y + 12 = 0,\ M_0(0,1,-1).$

6.24 $\sqrt{x^2 + y^2} + z^2 - 3z = 3,\ M_0(4,3,1).$

6.25 $x^2 + 2y^2 + 3z^2 = 59,\ M_0(3,1,4).$

6.26 $x^2 + y^2 + z^2 - 2xy - 2xz - 2yz = 17,\ M_0(-2,-1,2).$

6.27 $x^3 + 3xyz - z^3 = 27,\ M_0(3,1,3).$

6.28 $\ln z = x + 2y - z + \ln 3,\ M_0(1,1,3).$

6.29 $2x^2 + 2y^2 + z^2 - 8xz - z + 6 = 0,\ M_0(2,1,1).$

6.30 $z^2 = xy - z + x^2 - 4,\ M_0(2,1,1).$

ИДЗ-2 **Приложения частных производных**

1 Найти уравнение касательной плоскости и нормали в точке $M_0(x_0; y_0; z_0)$ к поверхности Ω, заданной уравнением:

1.1 $x^2 + y^2 + z^2 + 6z - 4x + 8 = 0,\ M_0(2,1,-1)$.

1.2 $x^2 - 4y^2 + z^2 = -2xy, M_0(-2,1,2)$.

1.3 $x^2 + y^2 + z^2 - xy + 3z = 7,\ M_0(1,2,1)$.

1.4 $x^2 + y^2 + z^2 + 6y + 4x = 8,\ M_0(-1,1,2)$.

1.5 $2x^2 - y^2 + z^2 - 4z + y = 13,\ M_0(2,1,-1)$.

1.6 $x^2 + y^2 + z^2 - 6y + 4z + 4 = 0,\ M_0(2,1,-1)$.

1.7 $x^2 + z^2 - 5yz + 3y = 46,\ M_0(1,2,-3)$.

1.8 $x^2 + y^2 - xz - yz = 0,\ M_0(0,2,2)$.

1.9 $x^2 + y^2 + 2yz - z^2 + y - 2z = 2,\ M_0(1,1,1)$.

1.10 $x^2 + y^2 - z^2 - 2xz + 2x = z,\ M_0(1,1,1)$.

1.11 $z = x^2 + y^2 - 2xy + 2x - y,\ M_0(-1,-1,-1)$.

1.12 $z = -x^2 + y^2 + 2xy - 3y,\ M_0(1,-1,1)$.

1.13 $z = x^2 - y^2 - 2xy - x - 2y,\ M_0(-1,1,1)$.

1.14 $x^2 - 2y^2 + z + xz - 4y = 13,\ M_0(3,1,2)$.

1.15 $4y^2 - z^2 + 4xy - xz + 3z = 9,\ M_0(1,-2,1)$.

1.16 $z = x^2 + y^2 - 3xy - x + y + 2,\ M_0(2,1,0)$.

1.17 $2x^2 - y^2 + 2z^2 + xy + xz = 3,\ M_0(1,2,1)$.

1.18 $x^2 - y^2 + z^2 - 4x + 2y = 14,\ M_0(3,1,4)$.

1.19 $x^2 + y^2 - z^2 + xz + 4y = 4,\ M_0(1,1,2)$.

1.20 $x^2 - y^2 - z^2 + xz + 4x = -5,\ M_0(-2,1,0)$.

1.21 $x^2 + y^2 - xz + yz - 3x = 11,\ M_0(1,4,-1)$.

1.22 $x^2 + 2y^2 + z^2 - 4xz = 8,\ M_0(0,2,0)$.

1.23 $x^2 - y^2 - 2z^2 - 2y = 0,\ M_0(-1,-1,1)$.

1.24 $x^2 + y^2 - 3z^2 + xy = -2z,\ M_0(1,0,1)$.

1.25 $2x^2 - y^2 + z^2 - 6x + 2y + 6 = 0,\ M_0(1,-1,1)$.

1.26 $x^2 + y^2 - z^2 + 6xy - z = 8,\ M_0(1,1,0)$.

1.27 $z = 2x^2 - 3y^2 + 4x - 2y + 10,\ M_0(-1,1,3)$.

1.28 $z = x^2 + y^2 - 4xy + 3x - 15,\ M_0(-1,3,4)$.

1.29 $z = x^2 + 2y^2 + 4xy - 5y - 10,\ M_0(-7,1,8)$.

1.30 $z = 2x^2 - 3y^2 + xy + 3x + 1,\ M_0(1,-1,2)$.

2 Найти частные производные второго порядка и убедиться в равенстве смешанных производных для функции:

2.1 $z = e^{x^2 - y^2}$

2.2 $z = \arccos(4x - y)$.

2.3 $z = \mathrm{ctg}(x + y)$.

2.4 $z = \mathrm{arctg}(5x + 2y)$.

2.5 $z = \mathrm{tg}(x / y)$.

2.6 $z = \mathrm{arctg}(2x - y)$.

2.7 $z = \cos(xy^2)$.

2.8 $z = \ln(4x^2 - 5y^3)$.

2.9 $z = \sin(x^2 - y)$.

2.10 $z = e^{\sqrt{x - y}}$.

2.11 $z = \mathrm{arctg}(x + y)$.

2.12 $z = \arcsin(4x + y)$.

2.13 $z = \arcsin(x - y)$.

2.14 $z = \arccos(x - 5y)$.

2.15 $z = \arccos(2x + y)$.

2.16 $z = \sin\sqrt{xy}$.

2.17 $z = \mathrm{arcctg}(x - 3y)$.

2.18 $z = \cos(3x^2 - y^3)$.

2.19 $z = \ln(3x^2 - 2y^2)$.

2.20 $z = \mathrm{arctg}(3x + 2y)$.

2.21 $z = e^{2x^2 - y^2}$.

2.22 $z = \ln(5x^2 - 3y^4)$.

2.23 $z = \mathrm{ctg}(y / x)$.

2.24 $z = \mathrm{arcctg}(x - 4y)$.

2.25 $z = \mathrm{tg}\sqrt{xy}$.

2.26 $z = \ln(3xy - 4)$.

2.27 $z = \cos(x^2 y^2 - 5)$.

2.28 $z = \mathrm{tg}(xy^2)$.

2.29 $z = \sin\sqrt{x^3 y}$.

2.30 $z = \arcsin(x - 2y)$

3 Проверить, удовлетворяет ли указанному уравнению функция $u(x; y)$:

3.1 $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = 0,\ u = \dfrac{y}{x}$.

3.2 $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} = 3(x^3 - y^3),\ u = \ln\dfrac{y}{x} + x^3 - y^3$.

3.3 $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0,\ u = \ln(x^2 + (y+1)^2)$.

3.4 $y \dfrac{\partial^2 u}{\partial x \partial y} = (1 + y\ln x)\dfrac{\partial u}{\partial x},\ u = x^y$.

3.5 $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} = 2u,\ u = \dfrac{xy}{x + y}$.

3.6 $x^2 \dfrac{\partial^2 u}{\partial x^2} + y^2 \dfrac{\partial^2 u}{\partial y^2} = 0,\ u = e^{xy}$.

3.7 $a^2 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial y^2},\ u = \sin^2(x - ay)$.

3.8 $x^2 \dfrac{\partial^2 u}{\partial x^2} - y^2 \dfrac{\partial^2 u}{\partial y^2} = 0,\ u = \sqrt{\dfrac{y}{x}}$.

3.9 $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0,\ u = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$.

3.10 $a^2 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial y^2},\ u = e^{-\cos(x + ay)}$.

3.11 $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0,\ u = (x - y)(y - z)(z - x)$.

3.12 $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} = u,\ u = x\ln\dfrac{y}{x}$.

3.13 $y \dfrac{\partial u}{\partial x} - x \dfrac{\partial u}{\partial y} = 0,\ u = \ln(x^2 + y^2)$.

3.14 $x^2\frac{\partial u}{\partial x}-xy\frac{\partial u}{\partial y}+y^2=0,\ u=\frac{y^3}{3x}+\arcsin(xy)$.

3.15 $x^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+2xy=0,\ u=e^{xy}$.

3.16 $\frac{\partial^2 u}{\partial x\partial y}=0,\ u=\operatorname{arctg}\frac{x+y}{1-xy}$.

3.17 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ u=\ln(x^2+y^2+2x+1)$.

3.18 $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+u=0,\ u=\frac{2x+3y}{x^2+y^2}$.

3.19 $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=1,\ u=\sqrt{x^2+y^2+z^2}$.

3.20 $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u,\ u=(x^2+y^2)\operatorname{tg}\frac{x}{y}$.

3.21 $9\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ u=e^{-(x+3y)}\sin(x+3y)$.

3.22 $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0,\ u=xe^{y/x}$.

3.23 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ u=\operatorname{arctg}\frac{y}{x}$.

3.24 $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0,\ u=\operatorname{arctg}\frac{x}{y}$.

3.25 $\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2}=0,\ u=\ln(x+e^{-y})$.

3.26 $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0,\ u=\arcsin\frac{x}{x+y}$.

3.27 $\frac{1}{x}\frac{\partial u}{\partial x}+\frac{1}{y}\frac{\partial u}{\partial y}=\frac{u}{y},\ u=\frac{y}{(x^2-y^2)^5}$.

3.28 $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=\frac{x+y}{x-y},\ u=\frac{x^2+y^2}{x-y}$.

3.29 $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=\frac{2y}{u},\ u=\sqrt{2xy+y^2}$.

3.30 $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0,\ u=\ln(x^2-y^2)$.

4 Исследовать на локальный экстремум функции:

4.1 $z=y\sqrt{x}-2y^2-x+14y$.
4.2 $z=x^3+8y^3-6xy+5$.
4.3 $z=1+15x-2x^2-xy-2y^2$.
4.4 $z=1+6x-x^2-xy-y^2$.
4.5 $z=x^3+y^2-6xy-39x+18y+20$.
4.6 $z=2x^3+2y^3-6xy+5$.
4.7 $z=3x^3+3y^3-9xy+10$.
4.8 $z=x^2+y^2+xy+x-y+1$.
4.9 $z=4(x-y)-x^2-y^2$.
4.10 $z=6(x-y)-3x^2-3y^2$.
4.11 $z=x^2+xy+y^2-6x-9y$.
4.12 $z=(x-2)^2+2y^2-10$.
4.13 $z=(x-5)^2+y^2+1$.
4.14 $z=x^3+y^3-3xy$.
4.15 $z=2xy-2x^2-4y^2$.
4.16 $z=x\sqrt{y}-x^2-y+18x+3$.
4.17 $z=2xy-5x^2-3y^2+2$.
4.18 $z=xy(12-x-y)$.
4.19 $z=xy-x^2-y^2+9$.

4.20 $z = 2xy - 3x^2 - 2y^2 + 10.$

4.21 $z = x^3 + 9y^3 - 6xy + 1.$

4.22 $z = y\sqrt{x} - y^2 - x + 6y.$

4.23 $z = x^2 - xy + y^2 + 9x - 6y + 20.$

4.24 $z = xy(6 - x - y).$

4.25 $z = x^2 + y^2 - xy + x + y.$

4.26 $z = x^2 + xy + y^2 - 2x - y.$

4.27 $z = (x-1)^2 + 2y^2.$

4.28 $z = xy - 3x^2 - 2y^2.$

4.29 $z = x^2 + 3(y+2)^2.$

4.30 $z = 2(x+y) - x^2 - y^2.$

5 Найти наибольшее и наименьшее значения функции $z = z(x; y)$ на компакте $\overline{D}$, ограниченном кривыми:

5.1 $z = 3x + y - xy,\ \overline{D}: y = x, y = 4, x = 0.$

5.2 $z = xy - x - 2y,\ \overline{D}: x = 3, y = x, y = 0.$

5.3 $z = x^2 + 2xy - 4x + 8y,\ \overline{D}: x = 0, x = 1, y = 0, y = 2.$

5.4 $z = 5x^2 - 3xy + y^2,\ \overline{D}: x = 0, x = 1, y = 0, y = 1.$

5.5 $z = x^2 + 2xy - y^2 - 4x,\ \overline{D}: x - y + 1 = 0, x = 3, y = 0.$

5.6 $z = x^2 + y^2 - 2x - 2y + 8,\ \overline{D}: x = 0, y = 0, x + y - 1 = 0.$

5.7 $z = 2x^3 - xy^2 + y^2,\ \overline{D}: x = 0, x = 1, y = 0, y = 6.$

5.8 $z = 3x + 6y - x^2 - xy - y^2,\ \overline{D}: x = 0, x = 1, y = 0, y = 1.$

5.9 $z = x^2 - 2y^2 + 4xy - 6x - 1,\ \overline{D}: x = 0, y = 0, x + y - 3 = 0.$

5.10 $z = x^2 + 2xy - 10,\ \overline{D}: y = 0, y = x^2 - 4.$

5.11 $z = xy - 2x - y,\ \overline{D}: x = 0, x = 3, y = 0, y = 4.$

5.12 $z = \frac{1}{2}x^2 - xy,\ \overline{D}: y = 8, y = 2x^2.$

5.13 $z = 3x^2 + 3y^2 - 2x - 2y + 2,\ \overline{D}: x = 0, y = 0, x + y - 1 = 0.$

5.14 $z = 2x^2 + 3y^2 + 1,\ \overline{D}: y = \sqrt{9 - \frac{9}{4}x^2}, y = 0.$

5.15 $z = x^2 - 2xy - y^2 + 4x + 1,\ \overline{D}: x = -3, y = 0, x + y + 1 = 0.$

5.16 $z = 3x^2 + 3y^2 - x - y + 1,\ \overline{D}: x = 5, y = 0, x - y - 1 = 0.$

5.17 $z = 2x^2 + 2xy - \frac{1}{2}y^2 - 4x,\ \overline{D}: x = 0, y = 2x, x = 0.$

5.18 $z = x^2 - 2xy + \frac{5}{2}y^2 - 2x,\ \overline{D}: x = 0, x = 2, y = 0, y = 2.$

5.19 $z = xy - 3x - 2y,\ \overline{D}: x = 0, x = 4, y = 0, y = 4.$

5.20 $z = x^2 + xy - 2,\ \overline{D}: y = 4x^2 - 4, y = 0.$

5.21 $z = x^2 y(4 - x - y),\ \overline{D}: x = 0, y = 0, y = 6 - x.$

5.22 $z = x^3 + y^3 - 3xy,\ \overline{D}: x = 0, x = 2, y = -1, y = 2.$

5.23 $z = 4(x - y) - x^2 - y^2,\ \overline{D}: x + 2y = 4, x - 2y = 4, x = 0.$

5.24 $z = x^3 + y^3 - 3xy,\ \overline{D}: x = 0, x = 2, y = -1, y = 2.$

5.25 $z = x^2 + 2xy - y^2 - 4x,\ \overline{D}: x = 3, y = 0, y = x + 1.$

5.26 $z = 6xy - 9x^2 - 9y^2 + 4x + 4y,\ \overline{D}: x = 0, x = 1, y = 0, y = 2.$

5.27 $z = x^2 + 2xy - y^2 - 2x + 2y,\ \overline{D}: x = 2, y = x + 2, y = 0.$

5.28 $z = 2x^2 y - x^3 y - x^2 y^2,\ \overline{D}: x = 0, y = 0, x + y = 6.$

5.29 $z = 4 - 2x^2 - y^2,\ \overline{D}: y = 0, y = \sqrt{1 - x^2}.$

5.30 $z = 5x^2 - 3xy + y^2 + 4,\ \overline{D}: x = -1, x = 1, y = -1, y = 1.$

Литература

1 Демидович, В. П. Сборник задач и упражнений по математическому анализу [Текст] : учебное пособие для вузов / В. П. Демидович. – М. : Наука, 1977.

2 Кудрявцев, Л. Д. Краткий курс математического анализа [Текст] : учебник для вузов / Л. Д. Кудрявцев.– М. : Наука, Гл. ред. физ.-мат. лит., 1989.

3 Кудрявцев, Л. Д. Сборник задач по математическому анализу [Текст] : учебное пособие для вузов / под ред. Л. Д. Кудрявцева. – М. : Наука, Гл. ред. физ.-мат. лит., 1984.

4 Математический анализ в вопросах и задачах [Текст] : учебное пособие для студентов вузов в 2 ч. Ч. 2. Функции нескольких переменных / под ред. В. Ф. Бутузова. – М. : Высш. шк., 1988.

5 Привалов, И. И. Введение в теорию функций комплексного переменного [Текст] : учебное пособие для вузов / И. И. Привалов. – М. : Наука, 1977.

6 Сборник индивидуальных заданий по высшей математике [Текст] : учебное пособие для вузов / под ред. А. П. Рябушко. – Мн. : Выш. шк., 1991.

7 Тер-Крикоров, А. М. Курс математического анализа: учебное пособие для вузов / А. М. Тер-Крикоров, М. И. Шабунин – М. : Наука, Гл. ред. физ.-мат. лит., 1988.